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Design of Scaled Down Models for Stability of Laminated Plates

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This study investigates problems associated with design of scaled down models. Similitude theory is employed to develop the necessary similarity conditions. Scaling laws provide the relationship between a full-scale structure and its scale models and can be used to extrapolate the experimental data of a small, inexpensive, and easily tested model into design information for the large prototype. Both complete and partial similarity are discussed. Particular emphasis is placed on the cases of buckling of rectangular symmetric angle-ply laminated plates under uniaxial compressive and shear loads. This analytical study indicates that distorted models with a different number of layers, material properties, and geometries than those of the prototype can predict the behavior of the prototype with good accuracy.

Nomenclature

A_{ij}	= laminate extensional stiffnesses
a, b	= plate length and width
B_{ij}	= laminate coupling stiffnesses
D_{ij}	= laminate flexural stiffnesses
E_{ij}	= Young's moduli of elasticity
h	= total laminate thickness
K_{xx}, K_s	= nondimensional critical loads
M_x, M_y	= moment resultants
$\bar{N}_{xx}, \bar{N}_{yy}$	= in-plane applied normal loads
$\bar{N}_{xy}$	= in-plane applied shear load
$Q_{ij}, \bar{Q}_{ij}$	= lamina stiffness elements
R	= aspect ratio
t	= ply thickness
u, v, w	= reference surface displacements
λ_i	= scale factors
ν_{ij}	= Poisson's ratios

Subscripts

m	= model
p	= prototype

Introduction

USE of reinforced composites in lightweight aerospace structures has increased steadily over the years. The outstanding mechanical and physical properties of advanced composites provide the engineer with potential to optimize properties specific to application. The increasing use of laminated composite components for a wide variety of applications in aerospace, mechanical, and other branches of engineering requires extensive experimental evaluation of any new design. Since reinforced composite components require efficiency and wisdom in design, sophistication and accuracy in analysis, and numerous and careful experimental evaluations, there is a growing interest in small-scale model testing.[1]

A scaled-down (by a large factor) model, scale model, which can predict the structural behavior of the full-scale system, prototype, can prove to be an extremely beneficial tool. This possible development must be based on the existence of certain structural parameters that control the behavior of the structural system when acted upon by static and/or dynamic loads. If such structural parameters exist, a scaled-down replica can be built, which will duplicate the response of the full-scale system. The two systems are then said to be structurally similar. The term, then, that best describes this similarity is structural similitude.

Similitude theory is employed to develop the necessary similarity conditions (scaling laws). Scaling laws provide the relationship between a full-scale structure and its scale models and can be used to extrapolate the experimental data of a small, inexpensive, and easily tested model into design information for a large prototype. The difficulty of making completely similar scale models often leads to accept certain type of distortion from exact duplication of the prototype (partial similarity). Both complete and partial similarity are discussed. A parametric investigation of problems associated with designing small-scale models for cross-ply laminated wide beams[1] and plates[2] has been presented. The procedure consists of systematically observing the effect of each parameter and corresponding scaling laws. Then acceptable intervals and limitations for these parameters and scaling laws are discussed. In each case, a set of valid scaling factors and corresponding response scaling laws which accurately predict the response of prototypes from experimental models is introduced. Particular emphasis is placed on the cases of buckling of rectangular angle-ply laminated plates under uniaxial compressive and shear loads. This analytical study also indicates that distorted models with a different number of layers, material properties, and geometries than those of the prototype can predict the behavior of the prototype with good accuracy.

The objectives of the investigation described herein are twofold. The first is to derive the necessary similarity conditions in order to design a distorted model that accurately predicts prototype behavior including distortion in stacking sequence and number of plies (N) and ply-level and sublaminate-level scaling; distortion in material properties, E_{ij}, ν_{ij}, and ρ; and distortion in fiber orientation angle θ. The second is to evaluate the derived similarity conditions analytically.

In all of our work in this area we will restrict ourselves to linearly elastic material behavior. Therefore, scale effects are not present. Furthermore, it is assumed that all laminas have equal thickness t, and the laminates are free of damage (delaminations, matrix cracking, fiber breaks, etc.).

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In this study, we consider only the procedure that is based on the direct use of the governing equations. This method is more convenient than dimensional analysis, since the resulting similarity conditions are forced by the governing equations of the system.

Buckling of Symmetric Laminated Angle-Ply Rectangular Plates

Consider symmetric angle-ply $[(+\theta/-\theta/+\theta\cdots)_s]$ laminated plates ($B_{ij}=0$). The plates are subjected to in-plane normal and shear loads ($\bar{N}_{xx}, \bar{N}_{yy}, \bar{N}_{xy}$). The buckling loads of symmetric angle-ply rectangular plates are governed by[3]

$$D_{11}w^0_{,xxxx}+4D_{16}w^0_{,xxxy}+2\bar{D}_{12}w^0_{,xxyy}+4D_{26}w^0_{,xyyy}$$
$$+D_{22}w^0_{,yyyy}-\bar{N}_{xx}w^0_{,xx}-\bar{N}_{yy}w^0_{,yy}-2\bar{N}_{xy}w^0_{,xy}=0 \qquad (1)$$

where $\bar{D}_{12}=D_{12}+2D_{66}$

For a simply supported plate, the boundary conditions are at $x=0, a$

$$w^0=0, \qquad M_x=-D_{11}w^0_{,xx}=0 \qquad (2)$$

and at $y=0, b$

$$w^0=0, \qquad M_y=-D_{22}w^0_{,yy}=0 \qquad (3)$$

The solution of the form

$$w^0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (4)$$

satisfies all boundary conditions, but does not satisfy the buckling equation, Eq. (1). Use of the modified Galerkin procedure yields

$$\left(D_{11}\frac{m^4}{a^4}+2\bar{D}_{12}\frac{m^2n^2}{a^2b^2}+D_{22}\frac{n^4}{b^4}+\frac{\bar{N}_{xx}}{\pi^2}\frac{m^2}{a^2}\right)A_{mn}$$
$$=\frac{32mn}{\pi^2 ab}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left[\frac{(m^2+p^2)}{a^2}D_{16}+\frac{(n^2+q^2)}{b^2}D_{26}+\frac{\bar{N}_{xy}}{4\pi^2}\right]$$
$$\times A_{pq}Q_{mnpq} \qquad (5)$$

for $m, n=1,2,\ldots,\infty$; subject to the constraints $m\pm p=$ odd and $n\pm q=$ odd. where $R=a/b$, and $Q_{mnpq}=\{pq/[(m^2-p^2)(n^2-q^2)]\}$.

Applying similitude theory to Eq. (5) (For details see Ref. 1)

$$\lambda_{D_{11}}\frac{\lambda_m^4}{\lambda_a^4}\lambda_{A_{mn}}=\lambda_{\bar{D}_{12}}\frac{\lambda_m^2\lambda_n^2}{\lambda_a^2\lambda_b^2}\lambda_{A_{mn}}=\lambda_{D_{22}}\frac{\lambda_n^4}{\lambda_b^4}\lambda_{A_{mn}}=\lambda_{\bar{N}_{xx}}\frac{\lambda_m^2}{\lambda_a^2}\lambda_{A_{mn}}$$
$$=\frac{\lambda_m\lambda_n}{\lambda_a^3\lambda_b}\lambda_{(m^2+p^2)}\lambda_{D_{16}}\lambda_\phi=\frac{\lambda_m\lambda_n}{\lambda_a\lambda_b^3}\lambda_{(n^2+q^2)}\lambda_{D_{26}}\lambda_\phi$$
$$=\frac{\lambda_m\lambda_n}{\lambda_a\lambda_b}\lambda_{\bar{N}_{xy}}\lambda_\phi \qquad (6)$$

where $\lambda_{x_i}=x_{i_p}/x_{i_m}$ denotes the scale factor of parameter x_i and

$$\phi=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}A_{pq}Q_{mnpq} \qquad (7)$$

Note that m, n, p, and q are integers which depend on the number of terms needed to approximate well the buckling mode shape. By assuming the mode shape for the model and its prototype to be approximately the same, both model and prototype can be well approximated by the same number of terms in the series with the same contribution of terms to the buckling mode and, thus,

$$\lambda_m=\lambda_n=\lambda_p=\lambda_q=\lambda_{A_{mn}}=1 \qquad (8)$$

then

$$\lambda_{(m^2+p^2)}=\lambda_{(n^2+q^2)}=\lambda_\phi=1 \qquad (9)$$

Uniaxial Compression.

Assuming $\bar{N}_{xy}=0$, Eq. (6) yields the following scaling laws for uniaxial compression:

$$\lambda_{K_{xx}}=\frac{\lambda_{D_{11}}}{\lambda_{E_{22}}\lambda_h^3\lambda_R^2} \qquad (10)$$

$$\lambda_{K_{xx}}=\frac{\lambda_{\bar{D}_{12}}}{\lambda_{E_{22}}\lambda_h^3} \qquad (11)$$

$$\lambda_{K_{xx}}=\frac{\lambda_{D_{22}}}{\lambda_{E_{22}}\lambda_h^3}\lambda_R^2 \qquad (12)$$

$$\lambda_{K_{xx}}=\frac{\lambda_{D_{16}}}{\lambda_{E_{22}}\lambda_h^3\lambda_R} \qquad (13)$$

$$\lambda_{K_{xx}}=\frac{\lambda_{D_{26}}}{\lambda_{E_{22}}\lambda_h^3}\lambda_R \qquad (14)$$

where $K_{xx}=(\bar{N}_{xx}b^2/E_{22}h^3)$.

Applying similitude theory to boundary conditions Eqs. (2) and (3) does not yield any scaling laws. These conditions, Eqs. (10–14), which contain both response and structural geometric parameters, are necessary scaling laws for laminated plates subjected to uniaxial compression loads.

Shear Buckling

We now consider a simply supported plate which is subjected to an in-plane shear stress ($\bar{N}_{xy}$). Assuming $\bar{N}_{xx}=0$, Eq. (6) yields the following scaling laws:

$$\lambda_{K_s}=\frac{\lambda_{D_{11}}}{\lambda_{E_{22}}\lambda_h^3\lambda_R^3} \qquad (15)$$

$$\lambda_{K_s}=\frac{\lambda_{\bar{D}_{12}}}{\lambda_{E_{22}}\lambda_h^3\lambda_R} \qquad (16)$$

$$\lambda_{K_s}=\frac{\lambda_{D_{22}}}{\lambda_{E_{22}}\lambda_h^3}\lambda_R \qquad (17)$$

$$\lambda_{K_s}=\frac{\lambda_{D_{16}}}{\lambda_{E_{22}}\lambda_h^3\lambda_R^2} \qquad (18)$$

$$\lambda_{K_s}=\frac{\lambda_{D_{26}}}{\lambda_{E_{22}}\lambda_h^3} \qquad (19)$$

where $K_s=(\bar{N}_{xy}b^2/E_{22}h^3)$.

These conditions, Eqs. (15–19), contain both response and structural geometric parameters.

So far, the necessary scaling laws for different destabilizing loads ($\bar{N}_{xx}, \bar{N}_{xy}$) for angle-ply laminated plates have been established. In the following sections, the possibility of the existence of different models is discussed. The definition of theoretical values of load parameters are those given by buckling theory for the prototype and its models. However, the predicted values of load parameter of the prototype are those given by projecting the model theoretical buckling load using the corresponding scaling laws.

Complete Similarity

The necessary condition for complete similarity between the model and its prototype is that all scaling laws be satisfied simultaneously. Here, the procedure is conducted for the case of uniaxial compression. Similar arguments can be made for the vibration and shear buckling cases. For uniaxial compression complete similarity requires

$$\lambda_{D_{11}}\lambda_R^{-2}=\lambda_{\bar{D}_{12}}=\lambda_{D_{22}}\lambda_R^2=\lambda_{D_{16}}\lambda_R^{-1}=\lambda_{D_{26}}\lambda_R \qquad (20)$$

Following Tsai,[4] laminate flexural stiffnesses are

$$D_{11}, D_{12}, D_{22}, D_{66}=h^3/12(\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{Q}_{66}) \qquad (21)$$

$$D_{16}, D_{26}=h^3/12[(3N^2-2)/N^3](\bar{Q}_{16}, \bar{Q}_{26}) \qquad (22)$$

where $\bar{Q}_{ij}$ are transformed reduced stiffnesses.

$$\lambda_{D_{11}}, \lambda_{D_{12}}, \lambda_{D_{22}}, \lambda_{D_{66}} = \lambda_h^3\left(\lambda_{\bar{Q}_{11}}, \lambda_{\bar{Q}_{12}}, \lambda_{\bar{Q}_{22}}, \lambda_{\bar{Q}_{66}}\right) \tag{23}$$

$$\lambda_{D_{16}}, \lambda_{D_{26}} = \lambda_h^3\lambda_\beta\left(\lambda_{\bar{Q}_{16}}, \lambda_{\bar{Q}_{26}}\right) \tag{24}$$

where $\beta = [(3N^2 - 2)/N^3]$.

N is the number of plies and h is the total thickness. It is assumed that all plies have equal thickness. By substituting Eqs. (23) and (24) into the complete similarity conditions, Eq. (20), and by assuming that the model and prototype have the same material properties and fiber orientation, $\lambda_{\bar{Q}_{ij}} = 1$. Then,

$$\lambda_R^{-2} = 1 = \lambda_R^2 = \lambda_\beta\lambda_R^{-1} = \lambda_\beta\lambda_R \tag{25}$$

These equalities, Eq. (25), are satisfied if

$$\lambda_\beta = \lambda_R = 1 \tag{26}$$

The simplest way to satisfy $\lambda_\beta = 1$ is to choose the same number of plies for model and prototype ($N_p = N_m$). By inspection it can be seen that these conditions, Eq. (20), are independent of ply thickness, and they only depend on material properties and number of plies. So, two plates with different ply thickness, $\lambda_t \neq 1$, but the same material properties and stacking sequences [i.e., $(+\theta/-\theta)_s$ and $(+\theta_n/-\theta_n)_s$] satisfy Eq. (20). This is called ply-level scaling, and it is the easiest way to achieve complete similarity.

So far, we have shown that in the special case of ply-level scaling, complete similarity can be achieved. However, there are some constraints in designing the model. These constraints involve the geometry of the model, the model material, the number of plies, and the stacking sequence of laminates. Since this still appears to be restrictive, we allow the use of distortion in the design of the model.

Partial Similarity

Often complete similarity is difficult to achieve or even undesirable. When at least one of the similarity conditions cannot be satisfied, partial similarity is achieved. In this case, the model which has some relaxation in similarity conditions is called a distorted model. These relaxations in the relationship between two systems cause model behavior to be different from that of the prototype. Since each variable has a different influence on the response of the system, the resulting similarity conditions have a different influence. By understanding the effect of variables and similarity conditions over desired intervals, the similarity conditions which have the least influence can be neglected without introducing significant error.[5]

The choice of the "right" type of distortion is investigated as follows. In each case, all of the model parameters except one are chosen to be identical to its prototype. Then, the effect of this relaxation for a wide range of this parameter is investigated.

Using Eqs. (21) and (22), the scaling laws for the uniaxial compression case, Eqs. (10–14), can be expressed as

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{11}}\lambda_{E_{22}}^{-1}\lambda_R^{-2} \tag{27}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{12}}\lambda_{E_{22}} \tag{28}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{22}}\lambda_{E_{22}}^{-1}\lambda_R^2 \tag{29}$$

$$\lambda_{K_{xx}} = \lambda_\beta\lambda_{\bar{Q}_{16}}\lambda_{E_{22}}^{-1}\lambda_R^{-1} \tag{30}$$

$$\lambda_{K_{xx}} = \lambda_\beta\lambda_{\bar{Q}_{26}}\lambda_{E_{22}}^{-1}\lambda_R \tag{31}$$

Now different possibilities of distorted models are considered.

Distortion in the Number of Plies

At first we consider models which have the same material properties as their prototype but with a different number of plies ($N_p \neq N_m$) and stacking sequences. There are three ways to scale down the number of plies in a model: 1) ply-level scaling $[(+\theta_n/-\theta_n)_s]$, 2) sublaminate level scaling[6] $[(+\theta/-\theta)_{ns}]$, and 3) general reduction of plies. The ply-level scaling leads to complete similarity (as already discussed). But the two other methods yield partial similarity. Since the model and prototype have the same material properties, the scaling laws, Eqs. (27–31), are simplified as

$$\lambda_{K_{xx}} = \lambda_R^{-2} \tag{32}$$

$$\lambda_{K_{xx}} = 1 \tag{33}$$

$$\lambda_{K_{xx}} = \lambda_R^2 \tag{34}$$

$$\lambda_{K_{xx}} = \lambda_\beta\lambda_R^{-1} \tag{35}$$

$$\lambda_{K_{xx}} = \lambda_\beta\lambda_R \tag{36}$$

The condition depicted by Eq. (33) is independent of all parameters. Conditions depicted by Eq. (32) and Eq. (34) are only functions of the aspect ratio scale factor (λ_R). However, Eqs. (35–36) are functions of N_p, N_m, and λ_R. For $N_P > N_m$ then $0 < \lambda_\beta < 1$. The simplest model is a model which has the same aspect ratio as the prototype ($\lambda_R = 1$). With this assumption the response scaling laws reduce to two conditions.

$$\lambda_{K_{xx}} = 1 \tag{37}$$

$$\lambda_{K_{xx}} = \lambda_\beta \tag{38}$$

These conditions yield accurate results if $N_p = N_m$, which means that the scale model and prototype are the same. The only possible way to design an accurate model with different number of plies than prototype is to choose a different aspect ratio for model than that of prototype. Figures 1–3 present the percentage of discrepancy between theoretical (th) and predicted (pr) values of uniaxial compression (K_{xx}) and shear (K_s) loads for different models with different number of plies and aspect ratios. The percentage of discrepancy (disc) is defined as

$$\%\text{disc(th and pr)} = 100 \times \frac{|\text{theory} - \text{predicted}|}{\text{theory}} \tag{39}$$

In these cases, the prototype is a Kevlar®/epoxy square plate $(+45/-45)_{10s}$. For buckling analysis both Eqs. (32) and (34) can be used for designing a model. Figure 1 presents the accuracy of models for uniaxial compression when Eq. (34) is used as the design condition. Note that this occurs when $R_m < 1$. It is necessary to note that a similar result can be achieved by using Eq. (32). In this case $R_m > 1$ leads to a model that is capable of predicting the prototype

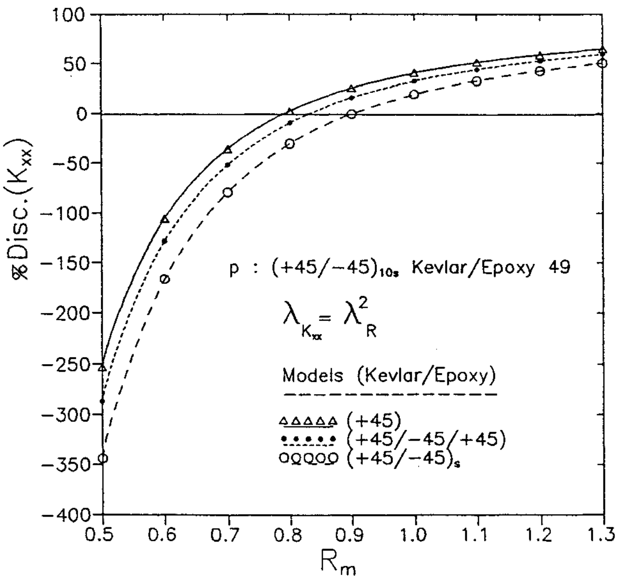

Fig. 1 Percentage of discrepancy (theoretical and predicted) of K_{xx} of the prototype when $N_m \neq N_p$ and $\lambda_{K_{xx}} = \lambda_R^2$.

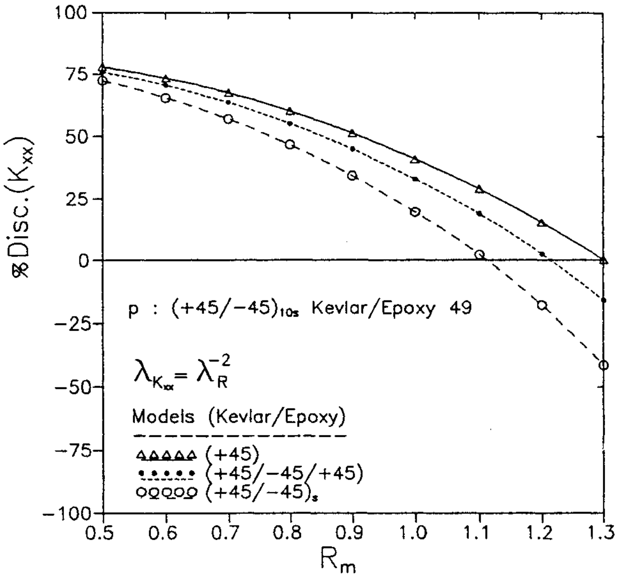

Fig. 2 Percentage of discrepancy (theoretical and predicted) of K_{xx} of the prototype when $N_m \neq N_p$ and $\lambda_{K_{xx}} = \lambda_R^{-2}$.

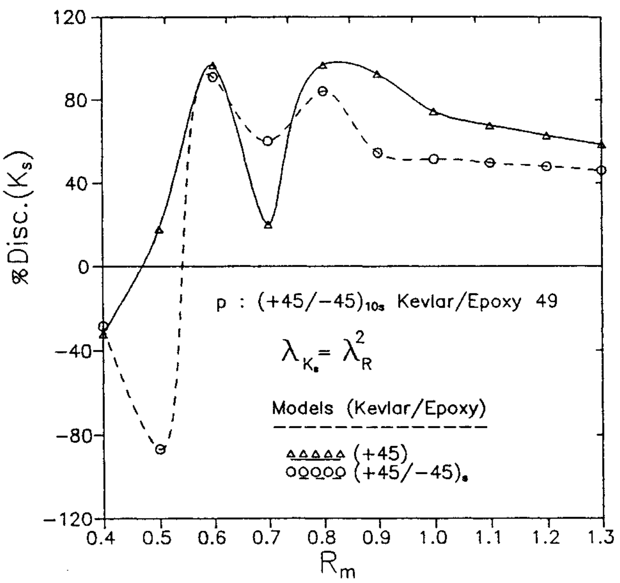

Fig. 3 Percentage of discrepancy (theoretical and predicted) of K_s of the prototype when $N_m \neq N_p$ and $\lambda_{K_s} = \lambda_R^{2}$.

behavior accurately. As the number of plies of the model increases, the necessary aspect ratio for the model R_m increases. In all of these cases response scaling laws involving λ_β do not yield good design conditions.

Distortion in Material Properties

Now we consider distortion in model material. Both isotropic materials (which include metals and plastics) and fiber reinforced composites are considered. In all of these cases the prototype is an angle-ply laminated plate. For the composite model, model and prototype have the same stacking sequence, number of plies $[(+45/-45)_{10s}]$, and aspect ratio. The scaling laws, Eqs. (27–31), are simpli-fied as

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{11}} \lambda_{E_{22}}^{-1} \tag{40}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{12}} \lambda_{E_{22}} \tag{41}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{22}} \lambda_{E_{22}}^{-1} \tag{42}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{16}} \lambda_{E_{22}}^{-1} \tag{43}$$

$$\lambda_{K_{xx}} = \lambda_{\bar{Q}_{26}} \lambda_{E_{22}}^{-1} \tag{44}$$

Figures 4 and 5 present theoretical and predicted buckling loads of the prototype and theoretical loads of the models for some typical composite materials. For the Kevlar/epoxy prototype most of the materials considered can be used as the model material or vice versa. In all of these cases, the scaling law described by Eq. (42) yields the best accuracy.

Since plastics are used extensively in experimental studies of the behavior of structures, the possibility of a plastic model or, in general, a model with isotropic material is considered. For isotropic materials, the assumption of $\lambda_R = 1$ yields a model which cannot predict accurately the behavior of the prototype. By choosing R_m as a design parameter we are able to find isotropic models which yield excellent accuracy. Figures 6 and 7 present the percentage of discrepancy between theoretical and predicted values of uniaxial compression (K_{xx}) and shear (K_s) loads for models made of isotropic materials as a function of model aspect ratios (R_m).

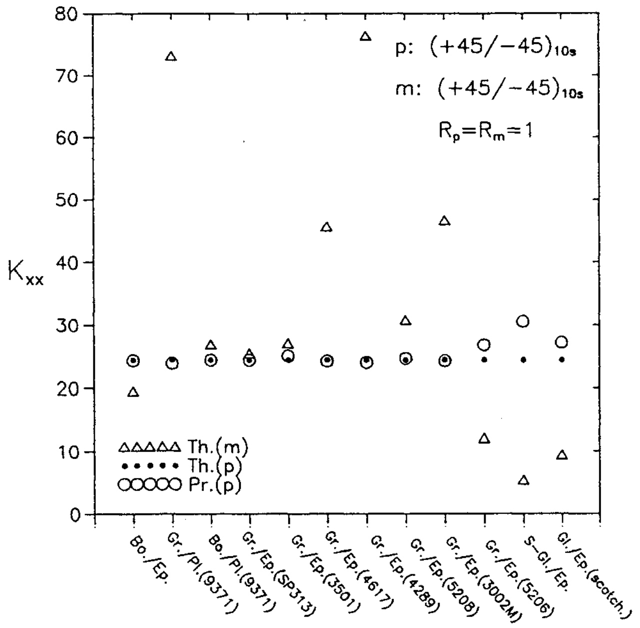

Fig. 4 Predicted and theoretical K_{xx} of the prototype using composite models.

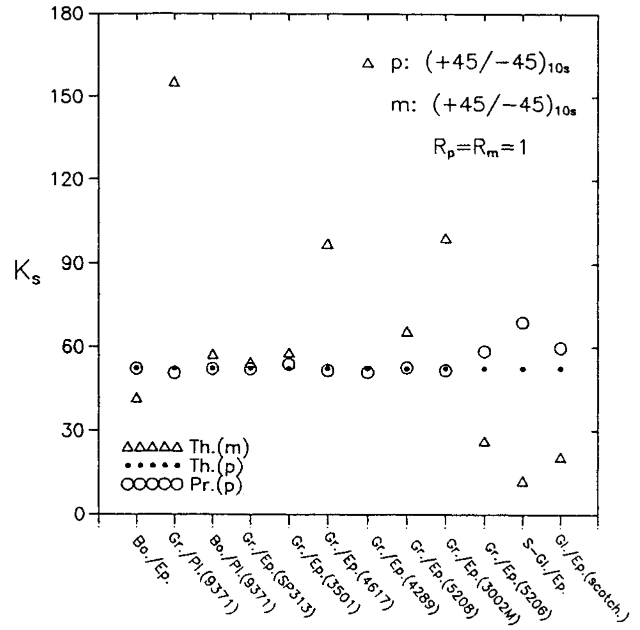

Fig. 5 Predicted and theoretical K_s of the prototype using composite models.

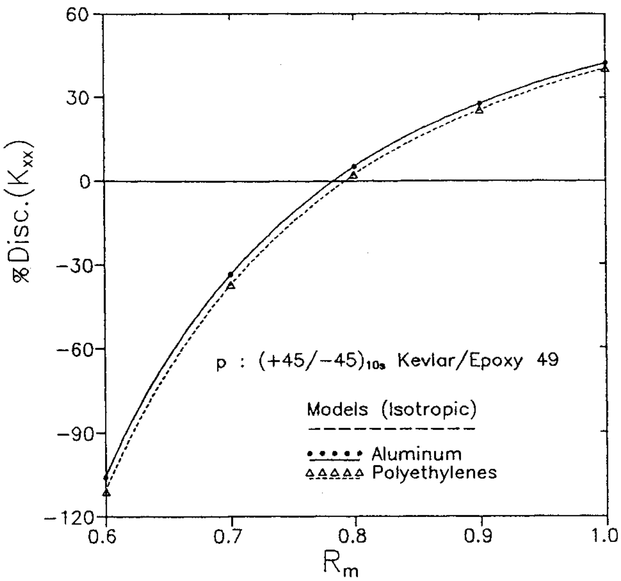

Fig. 6 Predicted and theoretical K_{xx} of the prototype using isotropic models.

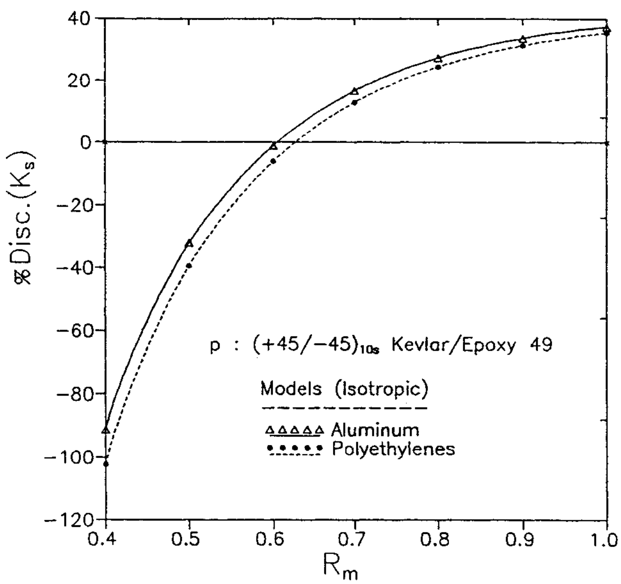

Fig. 7 Predicted and theoretical K_s of the prototype using isotropic models.

Discussion

The importance of employing small-scale models in designing advanced composite structures has been gaining momentum in recent years. With a view to better understanding the applicability of these models in designing laminated composite structures, an analytical investigation was undertaken to assess the feasibility of their use. Employment of similitude theory to establish similarity among structural systems can save considerable expense and time, provided the proper scaling laws are found and validated. In this study the limitation and acceptable interval of all parameters and corresponding scale factors are investigated. The results presented herein indicate that, for elastic response of an angle-ply rectangular plate based on structural similitude, a set of scaling laws can be found to develop design rules for small-scale models. The resulting scaling laws are very sensitive to distortion of number of plies of the model. However, by choosing a proper scale factor for the aspect ratio (λ_R), an accurate model can be designed. When model and prototype have the same number of plies and stacking sequences, a wide range of materials can be used as model materials with excellent accuracy.

Conclusions and Recommendations

This study presents the applicability of small-scale models, especially distorted models, in analyzing the elastic behavior of angle-ply symmetric laminated plates. Establishment of similarity conditions, based on the direct use of the governing equations, is discussed, and their use in the design of models is presented. Both complete and partial similarity are discussed. Distorted models, based on partial similarity, are more practical, since relaxation of each similarity condition eliminates some restrictions on the model design.

The results presented herein indicate that, for elastic response of an angle-ply rectangular plate based on structural similitude, a set of scaling laws can be found to develop design rules for small-scale models.

Some recommendations for future research include 1) the extension of the present work to curved configurations (cylindrical, conical, etc.), 2) study of the effect of boundary conditions, 3) study of the effect of geometric imperfections for imperfection-sensitive configurations, and 4) initiation of an experimental program for validation.

Acknowledgments

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